A Block Cipher Based Pseudo Random Number Generator Secure against Side-Channel Key Recovery

Christophe Petit[1], François-Xavier Standaert[1],
Olivier Pereira[1], Tal G. Malkin[2], Moti Yung[2]

[1]UCL Crypto Group, Université catholique de Louvain.
[2] Dept. of Computer Science, Columbia University.

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  - Issue : partial information on the SECRET is leaked by physical media
  - By recovering many pieces of partial info, one can recover the whole secret key

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  - (Try to) remove them by electronic countermeasures (masking, noise addition, dual-rails,...)

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  - Case Study : Pseudo-Random Number Generator (PRNG)

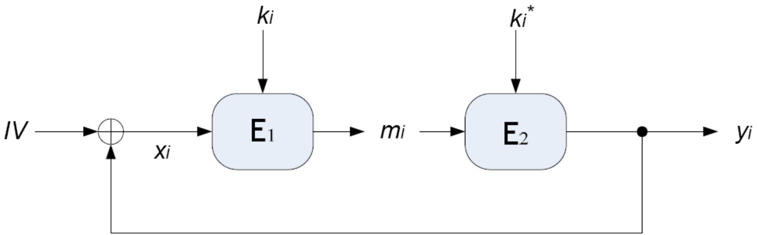
Case Study: PRNG

- Black-Box security (BB) : PRNG
- Grey-Box security (GB): prevent traditional SC cryptanalysis

Talk Overview

- Introduction
- PRNG
  - Construction
  - BB model & security
  - GB model & security
  - PRNG summary
- Conclusion and further work

Construction



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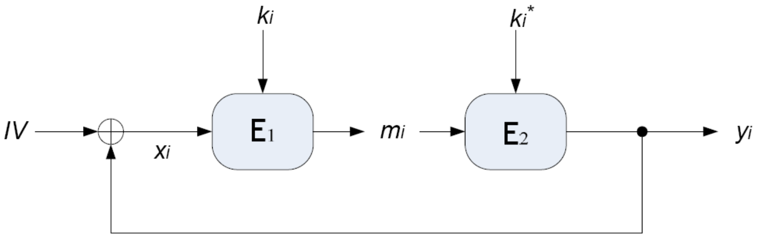

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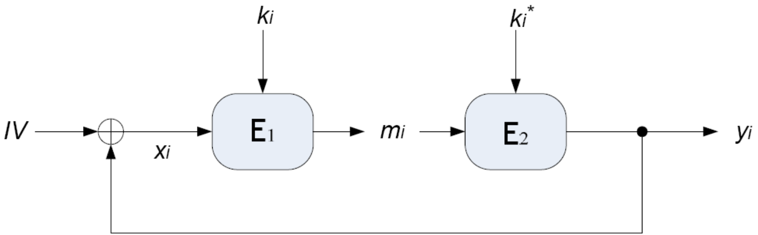

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- But (in GB) successive leakages allow recovering the whole secret.

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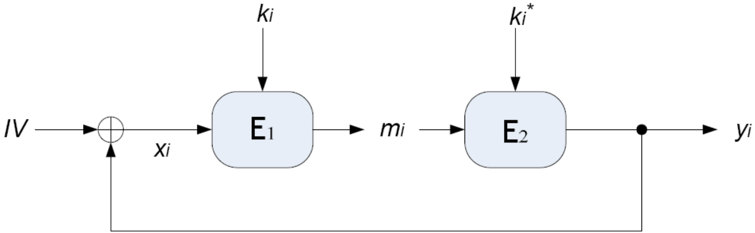

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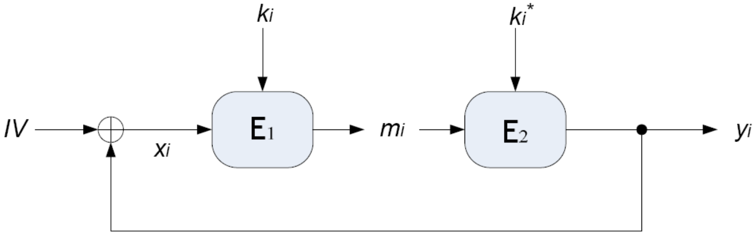

- So key update : $k_{i+1} = k_i \oplus m_i$ and $k^*_{i+1} = k^*_i \oplus m_i$
- Each running key k_i, k^*_i is used to encrypt *only* one message.

Black-Box Model

- Ideal cipher model $\mathsf{E} : \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{M}$
  - (Here $\mathcal{K} = \mathcal{M}$)
  - for each key $k \in \mathcal{K}$, the function $\mathsf{E}_k(\cdot) = \mathsf{E}(k, \cdot)$ is a random permutation on $\mathcal{M}$

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\mathbf{Succ}_{\mathsf{G,A}}^{\mathrm{prng}-1} &= \Pr[\mathsf{A}(\hat{k}) = 1 : \hat{k} \stackrel{R}{\leftarrow} \hat{\mathcal{K}}], \\
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  - G is a PRNG if for any A, $\mathbf{Adv}^{\mathrm{prng}}_{\mathrm{G,A}} \approx 0$.

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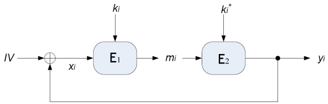

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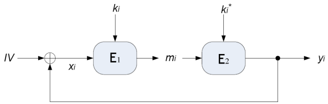

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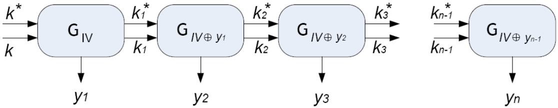

- For each $X \in \mathcal{M} = \mathcal{K}$, let $G_X : \mathcal{K} \times \mathcal{K} \rightarrow \mathcal{K} \times \mathcal{K} \times \mathcal{K}$

$$G_X(K, K^*) = (E_K(X) \oplus K, E_K(X) \oplus K^*, E_{K^*}(E_K(X))).$$

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- Security of a single round

  By definition,

$$\mathbf{Succ}^{\mathrm{prng}-0}_{\mathsf{G}_X,\mathsf{A}} = \Pr[\mathsf{A}(\hat{k}) = 1 : (k, k^*) \xleftarrow{R} \mathcal{K} \times \mathcal{K}; \hat{k} \leftarrow \mathsf{G}_X(k, k^*)]$$

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  - If there is A such that $\mathbf{Adv}^{\mathrm{prng}}_{\mathsf{G}^q,\mathsf{A}} > \epsilon$, then there is A′ such that $\mathbf{Adv}^{\mathrm{prng}}_{\mathsf{G},\mathsf{A}'} > \frac{\epsilon}{q}$ for one of the rounds

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- We show the available information does not permit recovering the secret

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- Side-channel key recovery adversary

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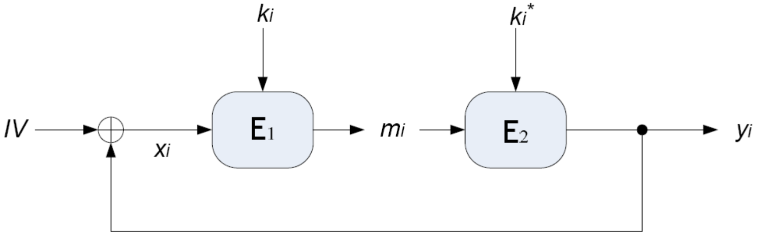

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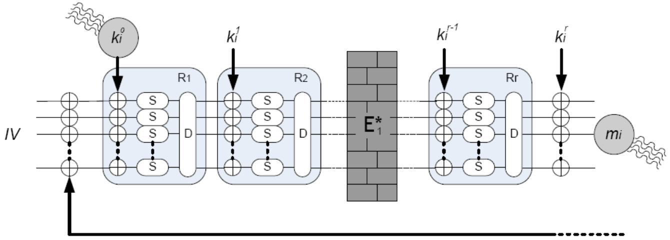

- Additional assumptions
  - Iterative BC, no key schedule
  - The adversary targets first round key $L(k_i) = L(k_i^0)$
  - Form of leakage functions : HW, GHW, NI

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- With observed leakages $\mathbf{l^q} = \{L(k_i), L(m_i)\}$ and relations $k_{i+1} = k_i \oplus m_i$, the best guess is

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- Goal : show that SR remains small as q increases

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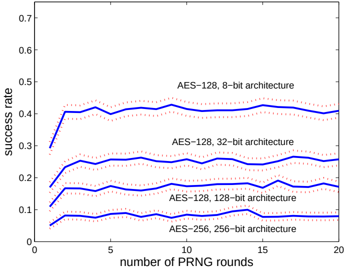
- (relevant in power consumption measures)
- In this case we compute : $\mathbf{Succ}^{\mathrm{sc-kr-}K_0}_{\mathrm{P}^q(K,K^*),\mathrm{A}} = \frac{n+1}{2^n}$
- High security, independently of q

Noisy Identity Leakages

- Here the above formulae are hard to evaluate analytically $\longrightarrow$ Monte-Carlo simulations



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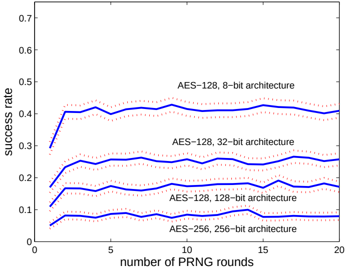

- $\mathbf{Succ}^{\text{sc-kr-K}}_{\text{AES256,A}} \simeq (0.08)^{32} = 2^{-116}$

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  With other countermeasures, leakages on more rounds means better attack

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- Need of theoretical framework for SC
  - unify BB and GB...
  - define physical primitives
  - compose primitives

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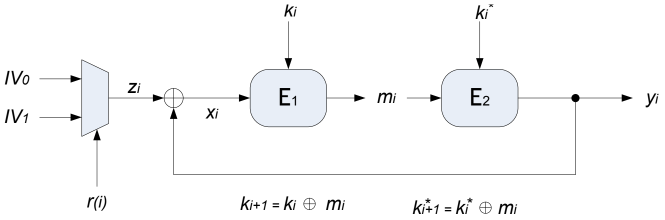

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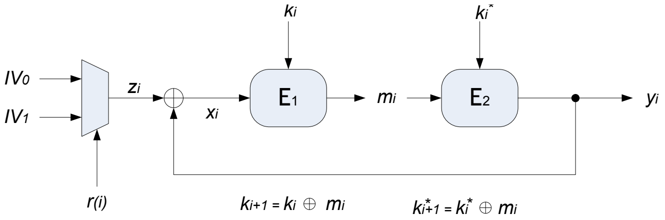

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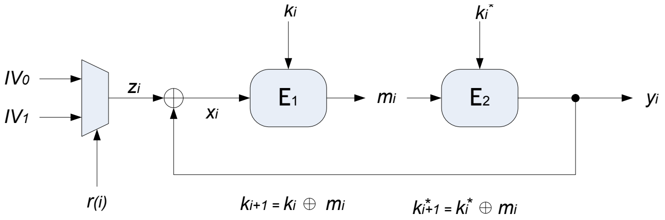

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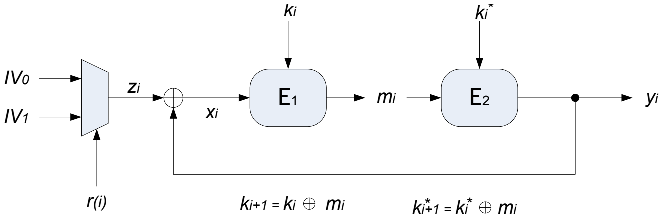

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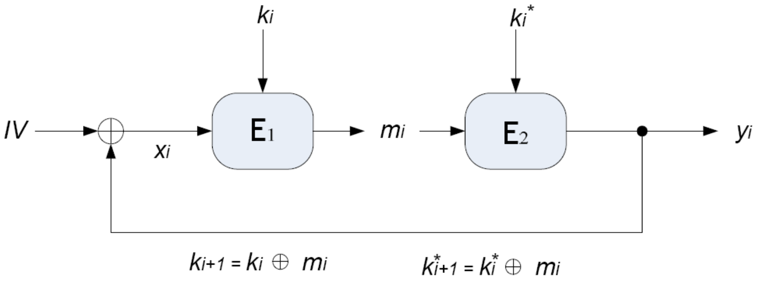

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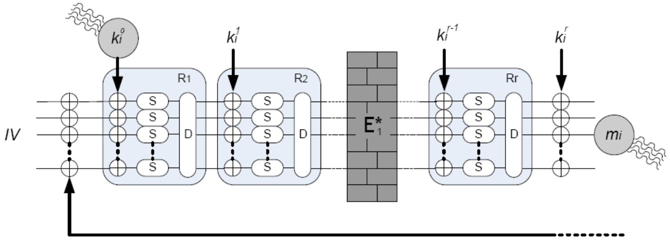

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  - Form of leakage functions : HW, GHW, NI
  - We suppose Bayesian adversary

Discussion about Grey-Box assumptions

- Many assumptions
  - make the proofs cleaner...
  - ...but are not essential.
- Relaxations $\longrightarrow$ same qualitative conclusions
  - key schedule $\longrightarrow$ adapt the leakage model $L(k_i)$
  - targeting not only the first iteration of the PRNG
    $\longrightarrow$ may increase SR, but qualitative results remains